Received: *April 18, 2000*, Accepted: *July 17, 2000*

Hyperbolic spaces in string and M-theory

Alexandros Kehagias

Physics Dept. National Technical University
GR-157 73 Zografou Campus, Athens, Greece
E-mail: kehagias@mail.cern.ch

Jorge G. Russo

Departamento de Física, Universidad de Buenos Aires
Ciudad Universitaria, Pabellón I, 1428 Buenos Aires
E-mail: russo@df.uba.ar

Abstract: We describe string-theory and $d = 11$ supergravity solutions involving symmetric spaces of constant negative curvature. Many examples of non-supersymmetric string compactifications on hyperbolic spaces H_r of finite volume are given in terms of suitable cosets of the form H_r/Γ, where Γ is a discrete group. We describe in some detail the cases of the non-compact hyperbolic spaces F_2 and F_3, representing the fundamental regions of H_2 and H_3 under $\mathrm{SL}(2,\mathbb{Z})$ and the Picard group, respectively. By writing AdS as a U(1) fibration, we obtain new solutions where AdS_{2p+1} gets untwisted by T-duality to $\mathbb{R}\times \mathrm{SU}(p,1)/(\mathrm{SU}(p)\times \mathrm{U}(1))$. Solutions with time-dependent dilaton field are also constructed by starting with a solution with NS5-brane flux over H_3. A new class of non-supersymmetric conformal field theories can be defined via holography.

Keywords: M-Theory, Superstring Vacua, AdS-CFT Correspondance.

JHEP07(2000)027

Contents

JHEP07(2000)027

1. Introduction

Maximally symmetric spaces play an important role in supergravity [1] and in superstring theory [2], and they naturally arise as the near-horizon region of black brane geometries [3, 4]. Maximally symmetric spaces (with no more than one time dimension) are the de Sitter dS_p and anti de Sitter AdS_p spaces, n-spheres S^n and r-dimensional hyperboloids H_r. Although supergravity solutions describing spheres and AdS spaces have been extensively investigated [1], not much has been done for solutions involving the de Sitter and hyperbolic geometries. There are several obvious reasons for this. De Sitter spaces always break supersymmetry, and they do not describe universes with zero cosmological constant. Hyperbolic spaces, such as the upper half r-plane, admit Killing spinors [5, 6, 7] but they have infinite volume. Therefore, they do not seem useful for describing internal spaces in string compactifications. Finite volume examples can be constructed by cosets of the form H_r/Γ, where Γ is a discrete subgroup of the isometry group of H_r [8, 9, 10]. However, dividing by Γ breaks all supersymmetries (see section 3).

We shall show that there is a large class of new string compactifications involving H_r or cosets H_r/Γ, which are regular (away possible orbifold points), where α' string corrections can be made very small for a sufficiently large radius of H_r. Among the possible applications of these solutions, we explore: a) the definition of new $\mathcal{N}=0$ conformal field theories in four dimensions via holography (see also [11, 12]); b) obtaining new models for dimensional reduction in non-compact spaces.

This paper is organized as follows. In section 2, we describe solutions of eleven-dimensional supergravity representing spaces which are direct products containing H_r factors and discuss the supersymmetry. Analogous solutions in type-II supergravity are also given. In section 3, we discuss finite-volume compactifications. In section 4 we compute the Newton gravitational potential for spaces having the hyperbolic 3-manifold H_3 as internal space. Because the Kaluza-Klein eigenvalue spectrum on H_3 is continuous with a mass gap, the Newton potential has a Yukawa-type behavior at large distances. Hyperbolic spaces naturally appear in theories with other spacetime signatures [13, 14, 15] (e.g. by Wick rotating AdS solutions to euclidean space) or in solutions with imaginary gauge fields, which can be interpreted as solutions with real gauge fields in new type-II* theories [14], where the signs of kinetic terms of RR gauge fields are reversed. In section 5, we consider two different approaches for obtaining new solutions in these theories. One of them consists in writing AdS_{2p+1} as a U(1) fibration and performing T-duality in a time-like direction. In section 6, we discuss the dual conformal field theories in compactifications that are direct product of anti de Sitter, hyperbolic manifolds of finite volume and spheres. Finally, we calculate in the appendix the Euler number of the space $\mathrm{SU}(2,1)/\mathrm{SU}(2)\times \mathrm{U}(1)$.

2. Hyperbolic manifolds in supergravity

2.1 Eleven-dimensional supergravity

The eleven-dimensional graviton multiplet contains the graviton g_{MN}, the antisymmetric three-form A_{MNK} and the gravitino Ψ_M $(M,N,\ldots=0,\ldots,10)$. The bosonic part of the eleven-dimensional supergravity lagrangian is

$$\mathcal{L}=\frac{1}{2\kappa_{11}^2}\sqrt{g}\left(R-\frac{1}{2\cdot 4!}F_{MNPQ}F^{MNPQ}\right)-\frac{1}{12^4}\epsilon^{M_1\ldots M_{11}}A_{M_1M_2M_3}F_{M_4\ldots M_7}F_{M_8\ldots M_{11}}\,. \tag{2.1}$$

A well-known solution to the equations of motions

$$R_{MN}=\frac{1}{12}\left(F_{MPQR}F_N{}^{PQR}-\frac{1}{12}g_{MN}F^2\right), \tag{2.2}$$

$$\nabla_M F^{MNPQ}=-\frac{1}{1152}\epsilon^{NPQR_1\ldots R_8}F_{R_1\ldots R_4}F_{R_5\ldots R_8}\,, \tag{2.3}$$

is provided by the Freund-Rubin ansatz for the antisymmetric field strength

$$\begin{aligned} F_{mnpq} &= 6m_0\,\epsilon_{mnpq} \qquad && \text{for } m,n,\ldots = 7,\ldots,10\,, \\ F_{MNPQ} &= 0\,, && \text{otherwise}\,. \end{aligned} \tag{2.4}$$

By substituting this ansatz into the field equations (2.2) we get

$$\begin{aligned} R_{\mu\nu} &= -6m_0^2 g_{\mu\nu}\,, \qquad \mu,\nu = 0,\ldots 6\,, \\ R_{mn} &= 12m_0^2 g_{mn}\,. \end{aligned} \tag{2.5}$$

The requirement of unbroken supersymmetry, i.e., the vanishing of the gravitino transformation

$$\delta\Psi_M = \nabla_M\epsilon - \frac{1}{288}\left(\Gamma_M{}^{PQRS} - 8\delta_M{}^P\Gamma^{QRS}\right)\epsilon F_{PQRS}\,, \tag{2.6}$$

for the ansatz (2.4), is equivalent to the existence of SO$(1,6)$ and SO(4) Killing spinors θ and η, respectively, which satisfy

$$\nabla_\mu\theta = \pm\frac{1}{2}m_0\gamma_\mu\theta\,, \tag{2.7}$$

$$\nabla_m\eta = \pm m_0\gamma_m\eta\,, \tag{2.8}$$

where $\gamma_\mu(\gamma_m)$ are SO$(1,6)$ (SO(4)) γ-matrices. Thus, although eq. (2.5) is solved for $Y^7\times X^4$ where Y^7 and X^4 are Einstein spaces of negative and positive curvature, respectively, only those spaces that admit Killing spinors obeying eqs. (2.7) and (2.8) are supersymmetric. The integrability condition of eqs. (2.7) and (2.8) are

$$W_{\mu\nu\rho\sigma}\gamma^{\rho\sigma}\theta = 0\,, \qquad W_{mnpq}\gamma^{pq}\eta = 0\,, \tag{2.9}$$

where $W_{\mu\nu\rho\sigma}$, W_{mnpq} are the Weyl tensors of Y^7, X^4, respectively. Thus, obvious supersymmetric examples for X^4 include the round four-sphere S^4 and its orbifolds S^4/Γ, where Γ is an appropriate discrete group [16]. For the Y^7 space one can take the anti-de Sitter space AdS_7, which preserves supersymmetry as well, and leads to the $AdS_7\times S^4$ vacuum of eleven-dimensional supergravity.

In addition to solutions of the form $AdS_7\times X^4$, there are other solutions to eq. (2.5) involving hyperbolic manifolds. The spaces $AdS_{7-r}\times H_r\times S^4$, where H_r, $r\geq 2$ is the r-dimensional upper-half space with metric:

$$ds^2 = dH_r^2 \equiv \frac{R_0^2}{x_0^2}(dx_0^2+dx_1^2+\cdots+dx_r^2)\,, \qquad R_0^2 = \frac{r}{6m_0^2}\,, \tag{2.10}$$

JHEP07(2000)027

solve eqs. (2.2), (2.3). Thus, the following backgrounds are all vacua of eleven-dimensional supergravity:

(i) $AdS_2 \times H_5 \times S^4$;

(ii) $AdS_2 \times H_2 \times H_3 \times S^4$;

(iii) $AdS_3 \times H_2 \times H_2 \times S^4$;

(iv) $AdS_3 \times H_4 \times S^4$;

(v) $AdS_4 \times H_3 \times S^4$;

(vi) $AdS_5 \times H_2 \times S^4$.

Neither of these backgrounds are supersymmetric. Indeed, they do not satisfy eq. (2.9). (Note that the product-space of conformally flat spaces is not conformally flat). However, although supersymmetry automatically guarantees the stability of the background, its absence does not necessarily implies instability. We recall for example the cases of $AdS_4 \times M^{pqr}$ [18] and $AdS_4 \times Q^{pqr}$ [19] compactifications of eleven-dimensional supergravity, which are stable for a certain range of p, q [20, 21]. A general result of the stability analysis in [20] was that, a generic $AdS_4 \times M_7$ compactification, where the Einstein space is a product of an n-dimensional manifold $M^n_{(1)}$ and a $7-n$-dimensional one $M^{7-n}_{(2)}$, $M_7 = M^n_{(1)} \times M^{7-n}_{(2)}$, is unstable. This is due to the fact that the stability of the AdS_4 vacuum depends on the eigenvalues of the Lichnerowicz operator Δ_L in the internal space M_7 acting on transverse, tracefree symmetric tensors h_{mn}. The background is supersymmetric if Δ_L is strictly positive (in fact Δ_L should be bigger than a multiple of m_0^2). However, for product spaces, Δ_L has a zero eigenvalue with eigenfunction $h_{mn} = \mathrm{diag}(g_{m_1n_1}, -\frac{n}{(7-n)} g_{m_2n_2})$, where $g_{m_1n_1}, g_{m_2n_2}$ are the metrics on $M^n_{(1)}, M^{7-n}_{(2)}$, respectively. Clearly, this eigenfunction corresponds to an expanding $M^n_{(1)}$ and a contracting $M^{7-n}_{(2)}$ with constant total volume of $M^n_{(1)} \times M^{7-n}_{(2)}$. Thus, the instability is due to volume-preserving metric deformations of the internal space.

All the (i)–(vi) vacua are clearly of the form described above, so it may appear that they are unstable if H_r is replaced by a finite volume coset H_r/Γ, as it will be described in the next section. However, this is not the case. Let us consider for example the case (v) involving AdS_4. Fixing the AdS radius, the corresponding radius of H_r is also fixed and instabilities can only be produced by metric perturbations on S^4 which preserve the Einstein metric. Such perturbations do not exist and although the background is of product form, it is stable. Of course, a definite statement for the stability of (i)–(vi) needs a case-by-case study of the spectrum. However, as the latter has not been studied yet, at least the potentially dangerous volume-preserving metric deformations of the internal space are absent.

Other non-supersymmetric compactifications can be obtained by replacing H_r in the spaces (i)–(vi) by any Einstein space of the same negative curvature. In particular, one has the solution

(vii) $AdS_3 \times \Sigma_4 \times S^4$

where Σ_4 is the coset space $\mathrm{SU}(2,1)/(\mathrm{SU}(2) \times \mathrm{U}(1))$. This space will appear again in section 5.1 (see also the appendix).

2.2 Hyperbolic manifolds in type-II supergravity

Simple type-IIB supergravity backgrounds with D3 brane charge and constant dilaton are the following ones:

(i) $AdS_3 \times H_2 \times S^5$;

(ii) $AdS_2 \times H_3 \times S^5$.

Like $AdS_5 \times S^2 \times S^3$, these spaces have no Killing spinors. In particular, in the case (ii), the metric is given by

$$ds^2_{10B} = \frac{R^2}{4z^2}(-dt^2 + dz^2) + \frac{R^2}{2v^2}(dx^2 + dy^2 + dv^2) + R^2 d\Omega_5^2 \,. \tag{2.11}$$

The 5-form self-dual field strength is the same as in the $AdS_5 \times S^5$ solution. The isometry group is $\mathrm{SO}(1,2) \times \mathrm{SO}(1,3) \times \mathrm{SO}(6)$. Although the solution is not supersymmetric, it is regular everywhere (being a direct product of Einstein spaces), and α' corrections can be made very small for sufficiently large radius R.

3. Finite volume cosets H_r/Γ

In the previous section we have seen some supergravity solutions involving factors AdS_p and H_r. The space H_r has infinite volume (with respect to the Poincaré metric). Thus there are no normalizable modes for any field in H_r, and the bulk as well as the boundary theories are both empty. Non-empty bulk and boundary field theories can be obtained by forming the coset H_r/Γ where Γ is a discrete subgroup of the isometry group of the upper-half plane. We shall now discuss some specific examples.

3.1 Killing spinors

The supergravity solutions involving the H_r manifolds discussed in section 2 are not supersymmetric. However, our analysis does not exclude that there could be more complicated backgrounds involving H_r, where some supersymmetries are unbroken. The Poincaré manifolds H_r admit Killing spinors (explicit expressions can be found in [6]), and a question of general interest is whether they survive under the orbifolding by the discrete group Γ. For this we may describe the upper-half space H_r as the hypersurface (taken for simplicity to be of constant curvature -1)

$$-X_0^2 + X_1^2 + \cdots + X_r^2 = -1 \,. \tag{3.1}$$

The isometry group of H_r is then clearly $\mathrm{SO}(1,r)$ and the group Γ is in general a subgroup of $\mathrm{SO}(1,r)$, which may or may not have fixed points. There are two cases to be discussed. If $r =$ even, then a Killing spinor is in the spinorial representation of $\mathrm{SO}(1,r-1)$. Then, if Γ is a subgroup of $\mathrm{SO}(1,r-1)$, but it is not a subgroup of $\mathrm{SO}(1,r-3)$, then there are no surviving Killing spinors. The latter exist if $\Gamma \subset \mathrm{SO}(1,r-3)$. However, in this case H_r/Γ will still be of infinite volume. Thus there are no finite volume cosets H_r/Γ, with $r = even$, with unbroken supersymmetries. In particular, any supergravity compactification involving finite volume cosets H_2/Γ and H_4/Γ will not preserve any supersymmetry.

The second case is when $r =$ odd. In this case, there are two Killing spinors on H_r in the spinorial representation of $\mathrm{SO}(1,r-1)$. These two Killing spinors are also Weyl spinors of the isometry group $\mathrm{SO}(1,r)$, so that they form an irreducible Dirac spinor of $\mathrm{SO}(1,r)$. If Γ is not a subgroup $\mathrm{SO}(1,r-1)$, then all supersymmetries are broken, while if Γ is a subgroup $\mathrm{SO}(1,r-1)$, then half of the supersymmetries survive.[1] In this case, it may seem that the space H_r/Γ can be of finite volume. In the particular case of H_3, we will explicitly see below that there is no choice of Γ that lead to a finite volume supersymmetric space.

3.2 Solutions of the form $AdS^5 \times H_2/\Gamma \times S^4$

Let us consider in first place the $AdS^5 \times H_2/\Gamma \times S^4$ solution of eleven-dimensional supergravity. The metric and four-form field strength are given by

$$\begin{aligned} ds_{11}^2 &= \frac{2R_0^2}{3z^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2) + \frac{R_0^2}{6v^2}(dx^2 + dv^2) + \frac{R_0^2}{4} d\Omega_4^2 \,, \\ \hat{F}_4 &= 6R_0^3 \Omega_4 \,, \end{aligned} \tag{3.2}$$

where $d\Omega_4^2$ is the metric on S^4 and we have taken the Poincaré metric for the upper half-plane H_2 ($v > 0$). A finite volume hyperbolic space can be obtained by forming the coset H_2/Γ, where Γ is a discrete subgroup of the isometry group $\mathrm{SL}(2,\mathbb{R})$ of the upper-half plane. Thus, Γ must be a fuchsian group, and the requirement of finiteness of the volume of H_2/Γ is equivalent to the fuchsian group Γ to be of the first kind. Examples are compact hyperbolic manifolds representing genus 2 Riemann surfaces.

A non-compact space of finite volume can be constructed by taking $\Gamma = \mathrm{SL}(2,\mathbb{Z})$, so that H_2/Γ is the fundamental domain F_2 of $\mathrm{SL}(2,\mathbb{Z})$. This case provides a rather

[1] The $r =$ odd case may be compared to the coset S^{2n+1}/Γ [17, 11, 12]. S^5/Γ is supersymmetric if $\Gamma \subset \mathrm{SU}(3), \mathrm{SU}(2)$, while is not supersymmetric if Γ is a maximal subgroup of $\mathrm{SU}(4)$.

simple setup for dimensional reduction, since there are no Kaluza-Klein modes associated with non-trivial quantum numbers in F_2. Modular functions which are eigenstates of the laplacian cannot be square-integrable, excepting the constant mode. Indeed, for a square-integrable wave function obeying $\Box\psi = \lambda\psi$, one can drop a boundary term in the following integration by parts

$$0 \le \int_{F_2} d\mu \partial\psi^* \partial\psi = -\int_{F_2} d\mu \psi^* \Box\psi = -\lambda \int_{F_2} d\mu \psi^* \psi \,, \qquad d\mu = \frac{dvdx}{v^2} \,,$$

which implies $\lambda \le 0$. But this contradicts the fact that the Laplace operator is negative in the fundamental domain (i.e. all eigenfunctions have $\lambda \ge 0$). Thus, scalar modes in $AdS_5 \times F_2 \times S^4$ will be of the form $\psi = \phi(AdS)Y(S^4)$, with no dependence on the F_2 coordinates.

3.3 The $AdS_4 \times F_3 \times S^4$ model

Let us now consider the $AdS_4 \times H_3/\Gamma \times S^4$ background. Here the supergravity solution is given by

$$\begin{aligned} ds_{11}^2 &= \frac{R_0^2}{2z^2}(-dt^2 + dx_1^2 + dx_2^2 + dz^2) + \frac{R_0^2}{3v^2}(dx^2 + dy^2 + dv^2) + \frac{R_0^2}{4} d\Omega_4^2 \,, \\ \hat{F}_4 &= 6R_0^3 \Omega_4 \,, \end{aligned} \tag{3.3}$$

In ref. [9], it was considered the case when H_3/Γ is the hyperbolic dodecahedron space, which is a compact hyperbolic manifold obtained by identifying opposite faces of a dodecahedron after a $3\pi/5$ rotation. More generally, we will show below that any finite volume space of the form H_3/Γ does not admit Killing spinors. We will mainly be interested in the case when H_3 is a non-compact hyperbolic manifold given by the fundamental region of the Picard group, which we describe next.

The space H_3 finds its natural description in terms of the quaternionic upper half 3-space (further details can be found in [22]). To define it, it is useful to recall a few known facts about quaternions, which are objects of the form

$$q = x + iy + ju + kv \,,$$

with x, y, u, v real, where

$$ij = k = -ji \,, \qquad jk = i = -kj \,, \qquad ki = j = -ik \,, \qquad i^2 = j^2 = k^2 = -1 \,.$$

The norm of a quaternion is given by $||q|| = \sqrt{x^2 + y^2 + u^2 + v^2}$. Quaternions can be represented in terms of Pauli matrices, so that SU(2) can be viewed as the unit quaternion.

The quaternionic upper half 3-plane is defined as follows:

$$H_3 = \{x + iy + kv | x, y, v \in \mathbb{R}, v > 0\} \tag{3.4}$$

JHEP07(2000)027

Thus the elements of H_3 are quaternions with j-coordinate equal to zero. It can be identified with the symmetric space $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)$. A matrix $g \in \mathrm{SL}(2,\mathbb{C})$ acts on an element q of the quaternionic upper half plane in the following way:

$$q' \equiv g(q) = (aq+b)(cq+d)^{-1}\,, \qquad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{3.5}$$

with $v' = v||cq+d||^{-2}$. The metric, volume element, and laplacian on H_3 are given by

$$\begin{aligned} ds^2 &= (dx^2+dy^2+dv^2)v^{-2}\,, \\ d\mu &= v^{-3}dxdydv\,, \\ \Delta &= v^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial v^2}\right) - v\frac{\partial}{\partial v}\,. \end{aligned} \tag{3.6}$$

The Picard group is defined to be

$$\Gamma = \mathrm{SL}(2,\mathbb{Z}[i]) = \left\{ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle|\, a,b,c,d \in \mathbb{Z}[i]\,,\ \det g = 1 \right\}, \tag{3.7}$$

with

$$\mathbb{Z}[i] = \{x+iy \,|\, x,y \in \mathbb{Z}\}$$

A fundamental domain for $\mathrm{SL}(2,\mathbb{Z}[i]) \setminus H_3$ in the quaternionic upper half plane is then

$$F_3 = \left\{ x+iy+kv \middle|\, |x| \le \frac{1}{2},\ 0 \le y \le \frac{1}{2},\ x^2+y^2+v^2 \ge 1,\ v > 0 \right\}. \tag{3.8}$$

The volume of the fundamental domain F_3 is [22]

$$\mathrm{Vol}(F_3) = \int_{F_3} d\mu = \frac{|D|^{3/2}\zeta_K(2)}{4\pi^2}\,, \tag{3.9}$$

where $\zeta_K(2)$ is the Dedekind zeta function of the number field $\mathbf{Q}(i)$.

Although non-supersymmetric, the space $AdS_4 \times F_3 \times S^4$ is an interesting setup for string compactification, in particular, for the construction of conformal field theories in $2+1$ dimensions (see section 6).

One can show that there is no supersymmetric compactification in supergravity involving F_3. Indeed, it is easy to see that no Killing spinor survives after dividing H_3 by $\mathrm{SL}(2,\mathbb{Z}[i])$. The isometry group of H_3 is $\mathrm{SO}(1,3) \cong \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. There are two Killing spinors which are Weyl spinors of each $\mathrm{SL}(2,\mathbb{R})$ factor. The Picard group $\mathrm{SL}(2,\mathbb{Z}[i])$ is essentially given by the subgroup $\mathrm{SL}(2,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$ of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. Therefore it does not leave invariant any of the Killing spinors. More generally, any finite volume example of the form H_3/Γ will not have Killing spinors, since the finite-volume condition implies that Γ must contain elements of both factors in $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$.

JHEP07(2000)027

4. Newton's law on H_3

Having described a number of string compactifications containing hyperbolic spaces, a question of interest (in view of the recent results of [23] for Newton's law in certain non-compact spaces) is whether such compactifications may be compatible with the observed Newton's law. In the case of compactification on a compact space X, there exist massive Kaluza-Klein modes of the four-dimensional graviton associated with the eigenvalues of the Laplace operator on X. In the static limit, these modes contribute to the Newton's law giving, in addition to the standard $1/r$ behavior of the massless graviton, Yukawa-type corrections. In particular, these are the expected corrections if the internal space is a compact hyperbolic manifold of the form H_r/Γ. What is the form of the corrections if the internal space is the non-compact, infinite volume hyperbolic manifold H_r? For non-compact spaces, the spectrum of the Laplace operator is continuous. For a continuum spectrum of eigenvalues, the gravitational potential is given by [24]

$$V(r) \sim \frac{1}{r}\, f(r)\,,$$

where

$$f(r) = \int dq\; e^{-qr} |\Psi_q(0)|^2\,,$$

and

$$-\nabla^2 \Psi_q(\rho) = q^2 \Psi_q(\rho)\,.$$

The standard $1/r$ behavior can still be recovered in some non-compact spaces [23].

Let us consider for simplicity $M^4 \times H_3$, where M^4 is the four-dimensional Minkowski space. Then, in "spherical" coordinates (ρ, θ, ϕ) for H_3 and metric

$$ds^2(H_3) = d\rho^2 + \sinh^2\rho \left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{4.1}$$

the eigenmodes are [25]

$$\Psi_{q\ell m}(\rho,\theta,\phi) = R^\ell_q(\rho) Y_{\ell m}(\theta,\phi)\,, \tag{4.2}$$

with a continuous spectrum of eigenvalues $q^2 = [1, \infty)$. The functions $Y_{\ell m}$ are spherical harmonics, and the radial eigenfunctions are the hyperspherical Bessel functions

$$R^\ell_q(\rho) = \frac{(-1)^{\ell+1}\sinh^\ell\rho}{\left(\prod_{n=0}^{\ell}(n^2+k^2)\right)^{1/2}} \frac{d^{\ell+1}\cos(k\rho)}{d(\cosh\rho)^{\ell+1}}\,, \qquad k^2 = q^2 - 1\,. \tag{4.3}$$

In the $\ell = 0$ sector (i.e. spherical modes with radial dependence only), we obtain

$$f(r) = \int_1^\infty dq\; \sqrt{q^2-1}\; e^{-qr} = \frac{K_1(r)}{r}\,, \tag{4.4}$$

where $K_1(r)$ is the modified Bessel function of the second kind. The gravitational potential in four dimensions is then

$$V(r) \sim \frac{K_1(r)}{r^2}\,. \tag{4.5}$$

At distances r much smaller than the radius of the hyperboloid (here taken to be 1 for simplicity), one has $V(r) \sim 1/r^3$. At large distances $r \gg 1$, the potential falls off exponentially, $V(r) = \sqrt{\pi/2r^5}\, e^{-r}$. This is expected, since there is a gap in the spectrum. As there is no any local minimum of $K_1(r)/r$, a $1/r$ approximated behavior cannot be obtained at any scale.

5. Hyperbolic spaces in type-IIA*/IIB* supergravities

5.1 AdS_{2p+1} as a U(1) fibration and untwisting

In ref. [26], a type-IIA solution of the form $AdS_5 \times CP_2 \times S^1$ was obtained by starting with $AdS_5 \times S^5$, writing the 5-sphere as a U(1) bundle over CP_2 (Hopf fibration) and performing T-duality along S^1. Now we make an analogous formal construction of a new type-IIA solution containing a hyperbolic space, by starting with a type-IIB solution describing AdS_3 or AdS_5, and performing a similar operation on the AdS part of the space. One can write AdS_{2n+1} as a U(1) bundle over a $2n$-dimensional base space, and then perform T-duality in the time direction, which untwists the fibration. As pointed out in [14], T-duality in a time-like direction does not connect type-IIA/IIB theory, but rather relates type-IIA to a type-IIB* and type-IIB to type-IIA*, obtained by reversing the signs of RR kinetic terms.

Let us illustrate this by first considering the case of the $AdS_3 \times S^3 \times T^4$ vacuum of type-IIB superstring theory, representing the geometry of the near-horizon D1-D5 brane system. The AdS_3 space is the hyperboloid

$$-X_0^2 + X_1^2 + X_2^2 - X_3^2 = -R^2\,, \tag{5.1}$$

or,

$$-W\bar{W} + Z\bar{Z} = -1\,, \qquad Z = X_1 + iX_2\,, \qquad W = X_0 + iX_3\,. \tag{5.2}$$

For simplicity, we have set the radius equal to one. The extension to an arbitrary radius is straightforward. By writing

$$Z = \frac{R}{(r^2/R^2-1)^{1/2}} e^{it}\,, \qquad W = \frac{r}{(r^2/R^2-1)^{1/2}} e^{i(t+\varphi)}\,,$$

one obtains the AdS_3 metric in the following form

$$ds^2 = \frac{dr^2 + r^2 d\varphi^2}{(1-r^2/R^2)^2} - R^2 \left(dt + \frac{r^2}{R^2-r^2} d\varphi\right)^2. \tag{5.3}$$

The existence of a U(1) symmetry and a U(1) fibration over a base H_2 is manifest in these coordinates. The fibration is trivial and can be turned into a non-trivial one after dividing H_2 by a discrete $\Gamma \subset \mathrm{SL}(2,\mathbb{R})$. The form of the metric eq. (5.3) makes also manifest the appearance of closed time-like curves. These can be avoided by unwrapping the time direction or, equivalently, by considering the universal covering space of AdS_3, which does not have any closed time-like curve. For the purpose of obtaining a new solution of type-II supergravity, we can formally perform a T-duality transformation along the S^1 compactified time direction in eq. (5.3). By using the standard T-duality transformation rules [27], we get the dual solution

$$d\hat{s}^2 = \frac{dr^2}{(1-\frac{r^2}{R^2})^2} + \frac{r^2}{(1-\frac{r^2}{R^2})^2}d\varphi^2 - R^2 dt^2\,, \quad B_{t\varphi} = -\frac{r^2}{1-\frac{r^2}{R^2}}\,, \quad e^{2\phi} = g^2 = \text{const.} \tag{5.4}$$

There are also components of the 2-form and 4-form field strength tensors given by

$$\hat{F}_2 = \frac{R^2\ r}{(1-r^2/R^2)^2}dr\wedge d\varphi\,, \qquad \hat{F}_4 = 2R^2\ dt\wedge\Omega_3\,. \tag{5.5}$$

Remarkably enough, the metric

$$d\sigma^2 = \frac{dr^2}{(1-r^2/R^2)^2} + \frac{r^2}{(1-r^2/R^2)^2}d\phi^2\,, \tag{5.6}$$

is the metric of the upper-half plane H_2. Thus, the T-dual of the $AdS_3\times S^3$ vacuum is the space $\mathbb{R}\times H_2\times S^3$, where we have unwrapped the time direction.

Let us now consider the anti de Sitter space AdS_5. In terms of complex coordinates, it is the hyperboloid:

$$-W\bar{W} + Z_1\bar{Z}_1 + Z_2\bar{Z}_2 = -R^2\,.$$

Write

$$W = \frac{R\ e^{it}}{(1-r^2/R^2)^{1/2}}\,, \qquad Z_1 = \frac{r}{(1-r^2/R^2)^{1/2}}e^{i(t+\frac{\chi+\varphi}{2})}\sin\frac{\theta}{2}\,,$$
$$Z_2 = \frac{r}{(1-\frac{r^2}{R^2})^{1/2}}e^{i(t+\frac{\chi-\varphi}{2})}\cos\frac{\theta}{2}\,.$$

The metric becomes

$$ds^2(AdS_5) = d\Sigma_4^2 - R^2(dt-A)^2\,, \tag{5.7}$$

where

$$d\Sigma_4^2 = \frac{dr^2}{(1-\frac{r^2}{R^2})^2} + \frac{1}{4}\frac{r^2}{(1-\frac{r^2}{R^2})}\left(d\theta^2+\sin^2\theta d\varphi^2\right) + \frac{1}{4}\frac{r^2}{(1-\frac{r^2}{R^2})^2}\left(d\chi-\cos\theta d\varphi\right)^2, \tag{5.8}$$
$$A = \frac{r^2}{2(R^2-r^2)}(d\chi-\cos\theta d\varphi)\,. \tag{5.9}$$

Now we make T-duality in time direction. This gives a new type-IIA* supergravity solution. After T-duality, we get

$$ds^2_{10A} = d\Sigma_4^2 - R^2 dt^2 + R^2 d\Omega_5^2\,, \qquad B_2 = R^2 dt \wedge A\,, \qquad e^{2\phi} = g^2\,. \tag{5.10}$$

There is also the 4-form field strength dA_3 with components in Σ_4 implied by the T-duality rules. Thus $AdS_5 \times S^5$ gets untwisted to a space $\mathbb{R} \times \Sigma_4 \times S^5$. The space Σ^4 has self-dual Weyl tensor and, in fact, it is the constant negative-curvature partner of CP_2, with Ricci tensor

$$\mathcal{R}_{\mu\nu}(\Sigma_4) = -\frac{6}{R^2} g_{\mu\nu}(\Sigma_4)\,.$$

Indeed, after $r \to ir$, we find that $d\Sigma_4^2 \to -ds^2(CP_2)$. It is not difficult to see that the space Σ_4 is the coset space $\mathrm{SU}(2,1)/\,\mathrm{SU}(2) \times \mathrm{U}(1)$. In general, the AdS_{2n+1} can be untwisted to $\Sigma_{2n} \times S^1$ where Σ_{2n} is the constant negative-curvature partner of CP_n with metric obtained from the latter after $r \to ir$ in the Fubini-Study metric of CP_n. Σ_{2n} is in fact the coset space $\mathrm{SU}(n,1)/\,\mathrm{SU}(n) \times \mathrm{U}(1)$. In the appendix we calculate the Euler number of this space.

5.2 Solutions with fluxes on H_r

A conformal field theory involving the upper half 3-space H_3 can be constructed as a WZW model based on the coset $\mathrm{SL}(2,\mathbb{C})/\,\mathrm{SU}(2)$. This can be combined with another CFT by direct product. In particular, it can be combined with the NS5-brane by summing both conformal sigma models (similar constructions have been done in [28]). To saturate the central charge, one needs to add a linear dilaton in the time direction. The construction leads to an NS-NS 2-form gauge field with imaginary components. By S-duality, one can convert it into a R-R 2-form with imaginary components, and thus into a solution of type-IIB* theory. The resulting conformal model is an exact solution of string theory to all orders in the α'-expansion.

These sigma models can also be constructed directly by brane intersections (the basic rules for constructing intersecting branes are in [29]). We first describe simpler string solutions based on the embedding H_3 in $\mathbb{R}^{10}$ (the extension to other H_r is straightforward). Let us consider spaces of the form $X_4 \times M_6$, with metric:

$$ds^2 = -dt^2 + t^2 dH_3^2 + ds_6^2\,, \qquad dH_3^2 = \frac{1}{z^2}(dx^2 + dy^2 + dz^2)\,, \tag{5.11}$$

where ds_6^2 describes the six-dimensional compact manifold M_6. The hyperbolic 3-manifold H_3 arises as constant time slices of X_4. The Riemann tensor for X_4 vanishes identically. To see this, one introduces new coordinates

$$U = \frac{t}{z}\,, \qquad X = \frac{tx}{z}\,, \qquad Y = t\frac{y}{z}\,, \qquad V = tz + \frac{t}{z}(x^2 + y^2)\,, \tag{5.12}$$

so that the metric for X_4 takes the form

$$ds_4^2 = -dUdV + dX^2 + dY^2 \,. \tag{5.13}$$

Thus eq. (5.12) provides the embedding of H_3 in a four-dimensional space-time. If $M_6 = T^6$, then the metric (5.11) is an exact solution of string theory, since the Riemann tensor identically vanishes. To leading order in α', one can take any Ricci-flat space M_6. From eq. (5.12) one gets

$$t^2 = UV - X^2 - Y^2 \,. \tag{5.14}$$

The condition that $t^2 \geq 0$ implies that the physical space-time is in the region $UV \geq X^2 + Y^2$.

To construct solutions with brane charges, a natural starting point is the NS5 brane, with flux on H_3. It is given by

$$ds_{10}^2 = dx_i^2 + f(t)(-\,dt^2 + t^2 dH_3^2)\,,\ dH_3^2 = \frac{1}{z^2}(dx^2 + dy^2 + dz^2)\,,\ i = 1, \ldots, 6 \tag{5.15}$$

$$e^{2\phi} = f(t)\,, \qquad idB = *df\,, \qquad f(t) = 1 + \frac{R^2}{t^2}\,. \tag{5.16}$$

This is a formal analog of the NS five brane solution, in which the 3-sphere has been replaced by a hyperbolic space. Note that the gauge field dB is imaginary. By S-duality, and redefining the RR 2-form by a factor i, this is converted into a solution of type-IIB* supergravity.

To have finite flux, the space H_3 can be replaced by the finite volume F_3 space, representing the fundamental region of the Picard group, as described previously (or by any coset H_3/Γ of finite volume).

By an appropriate rescaling of variables, one finds the "near horizon" solution:

$$ds_{10}^2 = dx_i^2 - d\tau^2 + R^2 dH_3^2\,, \qquad \phi = R^{-1}\tau\,, \qquad \tau = -\ln t\,. \tag{5.17}$$

This is a direct product of flat space with linear dilaton and a $\mathrm{SL}(2,\mathbb{C})/\,\mathrm{SU}(2)$ WZW model.

We now generalize the solution (5.15) by considering the intersection with an NS5 brane

$$\begin{aligned} ds_{10}^2 &= dx_1^2 + dx_2^2 + f(t)(\,-\,dt^2 + t^2 dH_3^2) + g(r)(dr^2 + r^2 d\Omega_3^2)\,, \\ e^{2\phi} &= f(t)g(r)\,, \qquad idB = *df(t) + *dg(r)\,, \qquad g(r) = 1 + \frac{R_0^2}{r^2}\,. \end{aligned} \tag{5.18}$$

Similarly, the NS5-brane can be combined with the solution (5.17), giving

$$\begin{aligned} ds_{10}^2 &= -d\tau^2 + dx_1^2 + dx_2^2 + R^2 dH_3^2 + g(r)(dr^2 + r^2 d\Omega_3^2) \\ e^{2\phi} &= e^{2\tau/R} g(r)\,. \end{aligned} \tag{5.19}$$

One can also add a fundamental string in the direction x_1 as follows:

$$\begin{aligned} ds^2_{10} &= h^{-1}(r)[-d\tau^2 + dx_1^2] + dx_2^2 + R^2 dH_3^2 + g(r)(dr^2 + r^2 d\Omega_3^2)\,, \\ e^{2\phi} &= e^{2\tau/R}\frac{g(r)}{h(r)}\,, \qquad h(r) = 1 + \frac{R_1^2}{r^2}\,. \end{aligned} \tag{5.20}$$

The near-horizon geometry is given by

$$\begin{aligned} ds^2_{10} &= \frac{r^2}{R_1^2}[-d\tau^2 + dx_1^2] + R_0^2\frac{dr^2}{r^2} + R^2 dH_3^2 + R_0^2 d\Omega_3^2\,, \\ e^{\phi} &= e^{\tau/R}\frac{R_0}{R_1}\,. \end{aligned} \tag{5.21}$$

This describes a background $AdS_3 \times S^3 \times H_3 \times S^1$, with a linear dilaton in the time direction.

By U-duality, one can construct different D-brane solutions with time dependence. In particular, starting with (5.15) and making S and T-dualities, one obtains solutions with Dp brane charge, with $p > 3$:

$$\begin{aligned} ds^2_{10B} &= f^{-1/2}(t)(dx_1^2 + \cdots + dx_{p+1}^2) + f^{-1/2}(t)[-dt^2 + t^2 dH_{8-p}^2] \\ e^{2\phi} &= f^{(3-p)/2}\,, \qquad \hat{F}_{8-p} = iR^{7-p}\hat{H}_{8-p}\,, \qquad f = 1 + \frac{R^{7-p}}{t^{7-p}}\,, \end{aligned} \tag{5.22}$$

where $\hat{H}_r$ is the volume form of H_r. In the case of $p = 3$, one gets near $t = 0$ a direct product of de Sitter space-time and the hyperboloid H_5, i.e. $dS_5 \times H_5$ (which can also be obtained from the $S^5 \times AdS_5$ solution by Wick rotation).

6. Discussion

The backgrounds described in section 2 can be used for the construction of new $\mathcal{N} = 0$ conformal field theories by holography. A version of the Maldacena dualities states that string/M theory on any space of the form $AdS_{d+1} \times X$ is equivalent to a conformal field theory in d dimensions. In particular, in the case of the $AdS_7 \times S^4$ compactification, the boundary theory is a CFT for the six-dimensional $\mathcal{N} = 2$ tensor multiplet [2], or a CFT for the six-dimensional $\mathcal{N} = 1$ vector multiplet in the $AdS_7 \times S^4/\Gamma$ background [16]. According to this, string (or M) theory compactified on spaces of the form $AdS_{d+1} \times (H_r/\Gamma) \times S^n$ defines a d dimensional conformal field theory with SO$(n+1)$ global symmetry. The correlation functions of the CFT are defined by the prescription [30, 31]

$$\left\langle e^{\int d^d x\, \phi_0(x)\mathcal{O}(x)} \right\rangle_{\rm CFT} \equiv Z_{\rm string}(\phi(\text{boundary}) = \phi_0(x))\,, \tag{6.1}$$

where $Z_{\rm string}$ is the string theory partition function computed with boundary values $\phi_0(x)$ of the string theory fields, which act as sources of CFT operators. As usual, in the classical supergravity approximation, it can be evaluated as $Z_{\text{string}} \cong \exp[-I_{SG}(\phi)]$, by solving the equations of motion of ϕ in the background for $AdS_{d+1} \times (H_r/\Gamma) \times S^n$, with the boundary condition $\phi(\text{boundary}) = \phi_0(x)$. Consider, in particular, the $d = 11$ supergravity solution $AdS_5 \times F_2 \times S^4$. This should be dual to a $d = 4$ non-supersymmetric CFT with SO(5) global symmetry group. Because the supergravity solution has M5-brane charge, one may expect the CFT to be related to the six-dimensional $(2, 0)$ or $(1, 0)$ conformal field theories. As explained in section 3, eigenfunctions of the Laplace operator on F_2 are not square-integrable, barring the constant mode. Thus Kaluza-Klein modes obeying the Laplace equation are constant in F_2. In the absence of supersymmetry, finding the field theory degrees of freedom is nevertheless difficult because the field theory is expected to be strongly coupled in the regime the supergravity description is valid.

Other interesting cases are the eleven-dimensional supergravity solution $AdS_4 \times F_3 \times S^4$, and the type-IIB solution $AdS_3 \times F_2 \times S^5$. Since the latter has D3 brane charge, it should be dual to some $\mathcal{N} = 0$, $d = 1 + 1$ conformal field theory related to $\mathcal{N} = 4$, $d = 3+1$ super Yang-Mills theory. The $AdS_4 \times F_3 \times S^4$ solution has M5 brane charge. The dual field theory should be a $d = 2 + 1$ conformal field theory related to the $6d$ $(2, 0)$ CFT, with an internal global symmetry group given by SO(5).

Acknowledgments

We would like to thank A. Tseytlin for useful comments, and S. Randjbar-Daemi for an important remark. A.K. is supported by a $\Gamma.\Gamma.$ E.T. grant No. $97/E\Lambda/71$. J.R. would like to thank the support of UBA, Conicet and Fundación Antorchas.

A. Euler number of Σ_4

In section 5 we have seen that the anti de Sitter spacetime AdS_{2p+1} can be represented as a U(1) fibration over a base space $\Sigma_{2p} = \mathrm{SU}(p, 1)/(\mathrm{SU}(p) \times \mathrm{U}(1))$. Here we present the calculation of the Euler number of Σ_4, which we expect will be of use for future applications. The space Σ_4 is a homogeneous space of the group SU(2, 1) and it is diffeomorphic to $\mathrm{SU}(2, 1)/\,\mathrm{SU}(2) \times \mathrm{U}(1)$. Its metric is given in eq. (5.8) and by using the standard SU(2) left-invariant one-forms σ_i, $i = 1, 2, 3$, $(d\sigma_1 = 2\sigma_2 \wedge \sigma_3, \ldots)$ on the S^3, we may express eq. (5.8) as

$$d\Sigma_4^2 = \frac{dr^2 + r^2\sigma_3^2}{(1 - r^2)^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{(1 - r^2)}\,, \tag{A.1}$$

where we have put $R = 1$. Recalling that the metric on CP_2 is

$$dCP_2^2 = \frac{dr^2 + r^2\sigma_3^2}{(1 + r^2)^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{(1 + r^2)}\,, \tag{A.2}$$

we see that the above metrics eqs. (A.1), (A.2) on $\Sigma_4 = \mathrm{SU}(2,1)/\,\mathrm{SU}(2)\times\mathrm{U}(1)$ and $CP_2 = \mathrm{SU}(3)/\,\mathrm{SU}(2)\times\mathrm{U}(1)$ are related by $r\to ir$. Note that although CP_2 is compact, Σ_4 is not and it has a boundary at $r=1$.

The Euler number $\chi(\Sigma_4)$ is calculated by the formula

$$\chi(M) = \frac{1}{32\pi^2}\int_M \epsilon^{abcd}R_{ab}\wedge R_{cd} - \frac{1}{32\pi^2}\int_{\partial M}\epsilon^{abcd}\left(2\theta_{ab}\wedge R_{cd} - \frac{4}{3}\theta_{ab}\wedge\theta_{ce}\wedge\theta_{ed}\right). \tag{A.3}$$

The first term is the standard bulk contribution whereas the last term is for the boundary corrections. A straightforward calculation for Σ_4 gives

$$\chi(\Sigma_4) = 3\left(1-\frac{1-2r_0}{(1-r_0^2)^2}\right) - \frac{3r_0^2+2r_0^4}{(1-r_0^2)^2} + \frac{1+2r_0^2}{(1-r_0^2)^2} = 1\,. \tag{A.4}$$

On the other hand, the Euler number of CP_2 is $\chi(CP_2)=3$ so the Euler numbers of CP_2 and Σ_2 differ by two units. This difference is due to the bold singularity that exists in the CP_2 space and it is absence in the case of Σ_4. Indeed, the CP_2 metric for $r\gg 1$ turns out to be

$$dCP_2^2 \sim \rho^{-4}\left(dr^2 + r^2\sigma_3^2\right) + \sigma_1^2 + \sigma_2^2\,, \tag{A.5}$$

and the surface $r\to\infty$ is a removable bolt singularity. The space Σ does not have any bolt. Since each bolt contributes two units to the Euler number, the Euler number of CP_2 will exceed the corresponding one of Σ_4 by two, as indeed happens.

References

[1] M.J. Duff, B.E. W. Nilsson and C.N. Pope, *Kaluza-Klein supergravity*, *Phys. Rep.* **130** (1986) 1.

[2] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[3] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[4] G.W. Gibbons and P.K. Townsend, *Vacuum interpolation in supergravity via super p-branes*, *Phys. Rev. Lett.* **71** (1993) 3754 [hep-th/9307049].

[5] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249.

[6] Y. Fujii and K. Yamagishi, *Killing spinors on spheres and hyperbolic manifolds*, *J. Math. Phys.* **27** (1986) 979.

[7] H. Lu, C.N. Pope and P.K. Townsend, *Domain walls from anti-de Sitter spacetime*, *Phys. Lett.* **B 391** (1997) 39 [hep-th/9607164];
H. Lu, C.N. Pope and J. Rahmfeld, *A construction of killing spinors on S^n*, *J. Math. Phys.* **40** (1999) 4518 [hep-th/9805151].

[8] J. Milnor, *Bull. Math. Soc.* **6** (1982) 9.

[9] I.Y. Aref'eva and I.V. Volovich, *Hyperbolic manifolds as vacuum solutions in Kaluza-Klein theories*, *Nucl. Phys.* **B 274** (1986) 619.

[10] N. Kaloper, J. March-Russell, G.D. Starkman and M. Trodden, *Compact hyperbolic extra dimensions: branes, Kaluza-Klein modes and cosmology*, hep-ph/0002001.

[11] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[12] A. Lawrence, N. Nekrasov and C. Vafa, *On conformal field theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[13] E. Cremmer, I.V. Lavrinenko, H. Lu, C.N. Pope, K.S. Stelle and T.A. Tran, *Euclidean-signature supergravities, dualities and instantons*, *Nucl. Phys.* **B 534** (1998) 40 [hep-th/9803259].

[14] C.M. Hull, *Timelike T-duality, de Sitter space, large-N gauge theories and topological field theory*, *J. High Energy Phys.* **07** (1998) 021 [hep-th/9806146].

[15] C.M. Hull and R.R. Khuri, *Worldvolume theories, holography, duality and time*, *Nucl. Phys.* **B 575** (2000) 231 [hep-th/9911082].

[16] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, *Membranes and fivebranes with lower supersymmetry and their AdS supergravity duals*, *Phys. Lett.* **B 431** (1998) 42 [hep-th/9803109];
AdS_6 interpretation of 5d superconformal field theories, *Phys. Lett.* **B 431** (1998) 57 [hep-th/9804006].

[17] M.R. Douglas and B.R. Greene, *Metrics on D-brane orbifolds*, *Adv. Theor. Math. Phys.* **1** (1998) 184 [hep-th/9707214];
M.R. Douglas, B.R. Greene and D.R. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[18] E. Witten, *Search for a realistic Kaluza-Klein theory*, *Nucl. Phys.* **B 186** (1981) 412;
L. Castellani, R. D'Auria and P. Fre, $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ *from $D = 11$ supergravity*, *Nucl. Phys.* **B 239** (1984) 610.

[19] R. D'Auria, P. Fre and P. van Nieuwenhuizen, *$N = 2$ matter coupled supergravity from compactification on a coset g/h possessing an additional killing vector*, *Phys. Lett.* **B 136** (1984) 347.

JHEP07(2000)027

[20] M.J. Duff, B.E. W. Nilsson and C.N. Pope, *The criterion for vacuum stability in Kaluza-Klein supergravity*, *Phys. Lett.* **B 139** (1984) 154.

[21] D.N. Page and C.N. Pope, *Which compactifications of $D = 11$ supergravity are stable?*, *Phys. Lett.* **B 144** (1984) 346;
Instabilities in englert type supergravity solutions, *Phys. Lett.* **B 145** (1984) 333.

[22] A. Terras, *Harmonics analysis on symmetric spaces and applications, 1, 2*, Springer-Verlag 1985.

[23] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[24] A. Kehagias and K. Sfetsos, *Deviations from the $1/R^2$ Newton law due to extra dimensions and α' corrections*, *Phys. Lett.* **B 472** (2000) 39 [hep-ph/9905417];
A. Kehagias, *On non-compact compactifications with brane worlds*, hep-th/9911134.

[25] N.J. Cornish and D.N. Spergel, *On the eigenmodes of compact hyperbolic 3-manifolds*, math.DG/9906017.

[26] M.J. Duff, H. Lu and C.N. Pope, *$AdS_5 \times S^5$ untwisted*, *Nucl. Phys.* **B 532** (1998) 181 [hep-th/9803061].

[27] E. Bergshoeff, C. Hull and T. Ortin, *Duality in the type-II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[28] G. Papadopoulos, J.G. Russo and A.A. Tseytlin, *Curved branes from string dualities*, *Class. and Quant. Grav.* **17** (2000) 1713 [hep-th/9911253].

[29] A.A. Tseytlin, *Composite BPS configurations of p-branes in 10 and 11 dimensions*, *Class. and Quant. Grav.* **14** (1997) 2085 [hep-th/9702163];
Composite BPS configurations of p-branes in 10 and 11 dimensions, *Class. and Quant. Grav.* **14** (1997) 2085 [hep-th/9702163].

[30] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[31] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].